IMT Atlantique
Bretagne-Pays de la Loire
École Mines-Télécom

PC1 – Let's practice rewriting

Langages et logique – ELU 610

Objectives

At the end of the activity, you should be capable of:

- writing and understanding a λ-term;
- compute in the λ-calculus (apply substitutions and reductions).

1 Basic syntax of the λ-calculus

Exercise 1 (*Parenthesis*)

▷ **Remove parenthesis as much as possible for the following λ-terms:**

1.1 $((\lambda y.(\lambda x.((yz)x)))(\lambda x.x))$

1.2 $(\lambda x.((\lambda y.((\lambda x.y)y))x))$

1.3 $((\lambda y.((\lambda z.A)(yy)))(\lambda x.(xx)))$ it is a metaterm, $A \in \Lambda_{\mathcal{X}}$

1.4 $(((ab)(cd))((ef)(gh)))$

Exercise 2 (*Tree representation*)

▷ **Give the tree representation of the following λ-terms:**

2.1 $\lambda x.(\lambda y.yy)zx$

2.2 $(\lambda y.yyy)(\lambda x.xx)$

2.3 $ux(yz)(\lambda v.vy)$

2.4 $(\lambda x.\lambda y.\lambda z.xz(yz))uvw$

2.5 $w(\lambda x.\lambda y.\lambda z.xz(yz))uv$

Exercise 3 (*Free variables*)

▷ **Give the set of free variables of the terms 1.1 to 2.5.**

Exercise 4 (*Substitution*)

▷ **Apply the following substitutions**

4.1 $[x \mapsto \lambda y.xy](\lambda y.x(\lambda x.x))$

4.2 $[y \mapsto \lambda v.vv](\lambda y.x(\lambda x.x))$

4.3 $[x \mapsto \lambda y.vy](y(\lambda v.xv))$

Exercise 5 (*Reduction*)

▷ **Gives all the possible sequences of reductions for the terms 1.1, 2.1, 2.4, then 2.2, 1.2 and 1.3.**

2 Data types in the λ-calculus

Exercise 6 (*Boolean*)

Let **T** and **F** be two combinators defined by $\lambda x.\lambda y.x$ and $\lambda x.\lambda y.y$.

▷ **Question 6.1:**
Show that the combinator $\mathit{If} = \lambda b.\lambda x.\lambda y.bxy$ **can be the usual if operator.**

▷ **Question 6.2:**
Give the combinators *Not*, *And* **and** *Or*

Exercise 7 (*Pairs*)

Pairs are defined by two functions *fst* and *snd*. A pair stores two values, the first being retrieved by *fst* and the second by *snd*. Let's denote a pair containing v_1 and v_2 by (v_1, v_2) then:

$$\begin{cases} \mathit{fst}\,(v_1, v_2) &= v_1 \\ \mathit{snd}\,(v_1, v_2) &= v_2 \end{cases}$$

▷ **Define** *fst* **and** *snd* **if we encode the pair** (v_1, v_2) **as** $\lambda f.f\,v_1\,v_2$.